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PERIODIC SOLUTIONS OF EVOLUTION EQUATIONS

K. Ezzinbi[1], T. Naito[2], N. Minh[3], J. Liu[4]

[1] Departement de Mathematiques
Faculte des Sciences Semlalia, B.P. 2390, Marrakech, Morocco [2] Department of Mathematics
University of Electro-Communications, Chofu, Tokyo 182-8585, Japan [3] Department of Mathematics
Hanoi University of Science, Hanoi, Vietnam [4] Department of Mathematics and Statistics
James Madison University, Harrisonburg, VA 22807, USA

Abstract. Consider the following evolution equations without delay or with finite or infinite delay in a general Banach space X,

$$u'(t) + A(t)u(t) = f(t, u(t)), \quad t > 0,$$

$$u'(t) + A(t)u(t) = f(t, u(t), u_t), \quad t > 0.$$

We will analyze some fixed point theorems and then see how they can be applied to derive periodic solutions for the above mentioned equations.
Keywords. Finite and Infinite Delays, Bounded and Periodic Solutions, Condensing Operators, Horn's, Sadovskii's, and Hale and Lunel's Fixed Point Theorems.
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1 INTRODUCTION

Let's first look at the following heat equation,

$$v_t(t, x) = v_{xx}(t, x) \text{ for } (t, x) \in (0, \infty) \times [0, 1], \tag{1.1}$$

with some initial and boundary conditions. If we define

$$u(t) = v(t, \cdot), \quad A = \partial_{xx} \text{ in } L^p(0, 1) \text{ (with some boundary conditions)}$$

then we obtain the following evolution equation

$$u'(t) = Au(t), \quad t > 0, \quad u(0) = u_0.$$

Based on this and other equations in applications, we generalize and then consider some *abstract evolution equations in infinite dimensional Banach spaces*, such as the following evolution equation without delay,

$$u'(t) + A(t)u(t) = f(t, u(t)), \quad t > 0, \quad u(0) = u_0,$$