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NONLINEAR MATHEMATICAL PROGRAMMING AND OPTIMAL CONTROL

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Abstract. The paper provides necessary conditions of optimality for a general minimization problem on a product space $Y \times X$ where the cost functional is a sum of a locally Lipschitz function and a convex, lower semicontinuous, proper function, while the constraints are of the form $Ay + Cu = B(y, u)$, with unbounded linear operators $A : D(A) \subset Y \to E$, $C : D(C) \subset X \to E$ and a Gâteaux differentiable mapping $B : Y \times X \to E$. One obtains a significant unification and extension of previous results. As applications one studies optimal control problems involving the wave operator and semilinear elliptic equations.
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1 Introduction

In this paper we deal with the nonlinear mathematical programming problem:

$$(P) \qquad \begin{array}{l} \text{Minimize } \{\Phi(y,u) + \Psi(y,u)\} \\ \text{subject to } Ay + Cu = B(y,u). \end{array}$$

Here, for real Banach spaces X, Y and E, $\Phi : Y \times X \to \mathbb{R}$ is a locally Lipschitz function, $\Psi : Y \times X \to \mathbb{R} \cup \{+\infty\}$ is a lower semicontinuous (l.s.c.), proper (i.e., $\Psi \not\equiv +\infty$) function with further properties specified later, $A : D(A) \subset Y \to E$ and $C : D(C) \subset X \to E$ are (possibly unbounded) linear operators and $B : Y \times X \to E$ is a (nonlinear) mapping.

The set of constraints is denoted by

$$M := \{(y,u) \in Y \times X : Ay + Cu = B(y,u)\}. \tag{1}$$

Throughout the paper we assume that $M \neq \emptyset$.

We are concerned with new necessary conditions of optimality for problem (P). These optimality conditions are achieved through various requirements. First, we obtain our necessary conditions in the case where the functional