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# ON KANTOROVICH INEQUALITY AND HÖLDER-MCCARTHY INEQUALITIES

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**Abstract.** We shall use the covariance-variance inequality as a tool to generalize Kantorovich inequality. We consider Hölder-McCarthy inequalities and related reverse inequalities. And finally the bound of a generalized Hölder-McCarthy inequality by recursion is given by way of the covariance-variance inequality. This is a continuous investigation in a different direction for Hölder-McCarthy inequalities from our previous article [5].

**Keywords.** Hölder-McCarthy inequality, Kantorovich inequality, covariance-variance inequality (c-v inequality), Löwner-Heinz inequality, bound of operator inequality.
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## 1. Notations and introduction

In what follows the capital letters mean bounded linear operators on a Hilbert space $H$ and the identity operator is denoted by $I$. We write $A \geq 0$ if $A$ is positive, i.e., $(Ax, x) \geq 0$ for all $x \in H$ and $A > 0$ if $A$ is positive and invertible. If $S$ and $T$ are selfadjoint, we write $T \geq S$ in case $T - S \geq 0$.

The following well known inequalities are crucial. McCarthy [6] proved the next two inequalities, called Hölder-McCarthy inequalities in literature, by using the spectral resolution of a positive operator and the Hölder inequality. More precisely, if $A \geq 0$, then, for any unit vector $x \in H$ and a real number $r$,

(**A**) $(A^r x, x) \leq (Ax, x)^r$ for $r \in [0, 1]$,

(**B**) $(Ax, x)^r \leq (A^r x, x)$ for $r \geq 1$.

Let us add one more inequality which appeared in [2, Theorem 1.5].

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