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QUENCHING-RATE ESTIMATE FOR A REACTION DIFFUSION EQUATION WITH WEAKLY SINGULAR REACTION TERM

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Abstract. We study the quenching problem for the reaction diffusion equation $u_t - u_{xx} = f(u)$ with Cauchy-Dirichlet data, in the case where the reaction term is singular at $u = 0$ in the sense that $\lim_{u\downarrow 0} f(u) = -\infty$. For $u > 0$ we take $f(u)$ to be smooth and to satisfy $(-1)^k f^{(k)}(u) < 0$; $k = 0, 1, 2$. Furthermore, we assume that $f(u)$ is weakly singular (in a neighborhood of the origin) in the sense that: $|u^n f^{(n)}(u)| = o(|f(u)|)$, as $u \downarrow 0$, $n = 1$ and $n = 2$.

We first show that for sufficiently large domains of x quenching occurs in finite time for these equations. The main result of this paper concerns the asymptotic behavior of the solution in a neighborhood of a quenching point. This result gives a uniform quenching-rate estimate in a region $|x| < C\sqrt{T-t}$ for the problem, when $(0, T)$ is a quenching point.
Keywords. Reaction-diffusion equation, quenching, quenching set, quenching rate, blowup
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1 Introduction

Consider the nonlinear diffusion problem

$$\begin{aligned} u_t - u_{xx} = f(u), \qquad x \in (-l, l), \quad t \in (0, T), \\ u(x, 0) = u_0(x), \quad x \in [-l, l], \\ u(\pm l, t) = 1, \quad t \in [0, T), \end{aligned} \tag{1}$$

where the initial function satisfies $0 < u_0(x) \leq 1$ and $u_0(\pm l) = 1$. Here T and l are positive constants. We assume that the reaction term $f(u)$ is singular at $u = 0$ in the sense that $\lim_{u\downarrow 0} f(u) = -\infty$. For $u > 0$ we take $f(u)$ to be smooth and to satisfy $(-1)^k f^{(k)}(u) < 0$; $k = 0, 1, 2$.

This type of reaction diffusion equation with singular reaction term arises in the study of electric current transients in polarized ionic conductors [17]. The problem can also be considered as a limiting case of models in chemical catalyst kinetics (Langmuir-Hinshelwood model) or of models in enzyme kinetics [7, 21].

The equation (1) has been extensively studied under assumptions implying that the solution $u(x, t)$ approaches zero in finite time. The reaction