

ON THE CONVERGENCE OF FINITE STEPS ITERATIVE SEQUENCES FOR ASYMPTOTICALLY NONEXPANSIVE MAPPINGS

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Abstract. The purpose of this paper is to study the weak and strong convergence of finite steps iterative sequences with mean errors to a common fixed point for a finite family of asymptotically nonexpansive mappings in Banach spaces. The results presented in this paper even in the cases of $N = 2$ or 3 are new.

Keywords. Finite steps iterative sequence, asymptotically nonexpansive mappings, iterative sequence with mean errors, common fixed point, Opial's condition, demi-closed principle, semi-compactness.

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1 Introduction and Preliminaries

Throughout this paper we assume that E is a real Banach space, C is a nonempty closed convex subset of E and $F(T)$ is the set of fixed points of T . We denote by $x_n \rightarrow x_0$ and $x_n \rightharpoonup x_0$ the strong convergence and weak convergence of $\{x_n\}$ converging to x_0 , respectively.

Recall that E is said to satisfy *Opial's condition* if, for each sequence $\{x_n\}$ in E , $x_n \rightharpoonup x$ implies that

$$\limsup_{n \rightarrow \infty} \|x_n - x\| < \limsup_{n \rightarrow \infty} \|x_n - y\|$$

for all $y \in E$ with $y \neq x$.

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