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NECESSARY AND SUFFICIENT VERTEX SOLUTIONS FOR ROBUST STABILITY ANALYSIS OF FAMILIES OF LINEAR STATE SPECE SYSTEMS

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Abstract. This paper addresses the issue of checking robust stability of families of linear state space systems which arises in many engineering applications such as switched systems and uncertain systems with real parameter variations. This problem can be eventually formulated as that of checking the stability of a convex combination of a set of Hurwitz stable 'vertex' matrices. Then the paper provides a complete solution to this problem in the form of a necessary and sufficient vertex solution. The proposed solution is significant in two ways in that it is a necessary and sufficient result and in that it requires only the vertex matrix information to infer the robust stability of the entire matrix family. Examples are included to illustrate the proposed algorithm. Finally, possible avenues of further computation reduction for the case of convex combinations induced by 'interval' parameters (labeled 'interval polytopes') are explored.
Keywords. Robust Stability;Convex Combination; Real Parameter Variation; Structured Uncertainty;Polytopes of Matrices; Extreme Point Results

1 Introduction

The problem of analyzing the stability of matrix families arises in many applications of systems and control theory [9]. The most common matrix family of interest is the family generated by a convex combination of arbitrary Hurwitz stable matrices. Consider the matrix family given by

$$\mathcal{A} \quad = \quad \left\{ A = \sum_{i=1}^{h} \alpha_i A^i, \alpha_i \geq 0, \sum \alpha_i = 1 \right\}, \tag{1}$$

where h is an integer and all the vertex matrices A^i are Hurwitz stable (i.e. have eigenvalues with negative real parts). Then the issue of research is to ascertain if all the matrices belonging to the above convex combination are also Hurwitz stable or not. The above problem formulation of checking the Hurwitz stability of a convex combination of *arbitrary* Hurwitz stable matrices, which is labeled as the 'General Polytope' problem in this paper, has not been researched even though it turns out to be a problem of interest in many applications such as linear switched systems [5].
However, the matrix family that has attracted considerable amount of research over the last two decades is the one arising in the area of linear state space systems with structured real parameters varying within given intervals. We label this family as the 'Interval Polytope' family. Very informative accounts of various aspects of this research are summarized in [9],[3],[4], [15].
For example, consider the linear state space description

$$\dot{x}(t) = A(q)x(t) \qquad q \in Q \tag{2}$$