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EXTERNAL STABILITY OF A DOUBLE INTEGRATOR WITH SATURATED LINEAR CONTROL LAWS

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Abstract. For a double integrator subject to input saturation, it is well-known that linear control laws can achieve global asymptotic stability. But a study of external stability for such a simple system reveals an unexpectedly rich nature. It is shown in this paper that external L_p stability for non-input-additive disturbance only holds for $p \leq 2$, but not for $p > 2$ no matter what linear control law is used. However, for input-additive disturbance, L_p stability holds for all $1 \leq p < \infty$. As a third result, we show that the double integrator system controlled by a saturating linear feedback is not input-to-state stable (ISS) even when all disturbances have their magnitudes restricted to be arbitrarily small. These results for the first time reveal that external stability of nonlinear systems is essentially different from that of linear systems. A fundamental discovery in this study is that the external stability of nonlinear systems cannot be separated from the internal state behavior.

Keywords. Input saturation, asymptotic stability, L_p stability, input-to-state stability, disturbance rejection.

1 Introduction

Recently there has been a lot of interest in the study of linear systems with input saturation. The two recent special issues [1, 11] have been devoted to this problem. The problem can be simply described as follows. Given the linear system with saturated input and external disturbance

$$\dot{x} = Ax + B\sigma(u) + w \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the input, $w \in \mathbb{R}^n$ is the disturbance, and $\sigma(\cdot)$ indicates a componentwise saturation function. Find a state feedback $u = f(x)$ such that the closed loop system

$$\dot{x} = Ax + B\sigma(f(x))$$