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ANALYSIS OF A MODEL OF THE NUTRIENT DRIVEN SELF-CYCLING FERMENTATION PROCESS

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Abstract. Self-cycling fermentation is a computer-aided process used for culturing microorganisms. Potential applications could include water purification, treatment of sewage, and the cleanup of toxic waste. We consider a model of growth of a single species in the fermenter, assuming a single limiting nutrient, with the level of this nutrient as the triggering factor. The model is formulated in terms of impulsive ordinary differential equations. The model predicts that either the system fails and the population of microorganisms essentially washes out, or more favourably, the fermenter cycles indefinitely, with one impulse per period, maintaining a positive, though oscillatory, number of cells. The predicted outcome is based on a threshold criterion that can be determined in advance in terms of biologically relevant parameters. An analytic expression for the cycle time is also derived. Using this expression, it is shown that the total yield over a specified time period depends on the choice of the optimal emptying/refilling fraction. A method for determining the optimal emptying/refilling fraction is given. The results are illustrated by means of numerical simulations.

Keywords. Self-cycling fermentation, impulsive differential equations, nutrient driven process, tolerance, moment of impulse, emptying/refilling fraction, cycle time, optimal yield.

AMS (MOS) subject classification: 34K45, 34K60, 92D25, 92D40, 62P12

1 Introduction

The development of the process of self-cycling fermentation (SCF) is described in Sheppard and Cooper [11], and a model similar to the one discussed here is given in Wincure, Cooper, and Rey [16].

Briefly, the process of self-cycling fermentation is a computer-controlled semibatch fermentation. A well-stirred tank containing fresh medium is inoculated with microorganisms that consume the nutrients in the broth. The microorganisms process the nutrient in order to grow and to reproduce. A

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