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A DIRECT METHOD OF OPTIMIZATION AND ITS APPLICATION TO A CLASS OF DIFFERENTIAL GAMES

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Abstract: A coordinate transformation approach is employed to deduce a direct, non-variational method to obtain extremizing solutions for some classes of integrals as well as optimal control and differential game problems.
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1 Introduction

Many problems in economics and engineering can be posed as dynamic optimization problems involving the extremization of an integral over a given class of functions subject to prescribed end conditions. These problems are usually addressed via the Calculus of Variations or the Maximum Principle of optimal control theory, applying necessary conditions to obtain candidate optimal solutions, and then assuring optimality via sufficient conditions if available. These methods are variational in that they employ the comparison of solutions in a neighborhood of the optimal one.

A different approach was first proposed in the 1960's and more recently expanded in [6]-[9]. This approach permits the direct derivation of global extrema for some classes of dynamic optimization problems without the use of comparison techniques. Instead, it employs coordinate transformations and the imposition of a functional identity. The direct method is readily applicable to a class of open-loop differential games, as shown in [5].

Finally, in order to enlarge the class of problems which can be treated by the proposed direct optimization method, a modification of the method is introduced.

2 Calculus of Variations

The Basic Problem

Given

$$I(y(\cdot)) := \int_a^b F[x, y(x), y'(x)]dx \tag{2.1}$$