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STABILITY THEORY FOR SET DIFFERENTIAL EQUATIONS

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To our dear friend Professor D. D. Siljak on his 70th birthday.

Abstract. The formulation of set differential equations has an intrinsic disadvantage that the diameter of the solution is nondecreasing as time increases and therefore the behavior of solutions, in some cases, do not match with the solutions of ordinary differential equations from which set differential equations can be generated. In this paper an approach is provided to remove the disadvantage.
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1 Preliminaries

Let $K(R^n)(K_c(R^n))$ denote the collection of all nonempty, compact (compact, convex) subsets of R^n. Define the Hausdorff metric

$$D[A,B] = \max\left[\sup_{x\in B} d(x,A), \sup_{y\in A} d(y,B)\right] \tag{1.1}$$

where $d[x,A] = \inf[d(x,y) : y \in A]$, A, B are bounded sets in R^n. We note that $K(R^n), (K_c(R^n))$, with the metric is a complete metric space.

It is known that if the space $K_c(R^n)$ is equipped with the natural algebraic operations of addition and nonnegative scalar multiplication, then $K_c(R^n)$ becomes a semilinear metric space which can be embedded as a complete cone into a corresponding Banach space [2, 14].

The Hausdorff metric (1.1) satisfies the following properties.

$$D[A+C, B+C] = D[A,B] \text{ and } D[A,B] = D[B,A], \tag{1.2}$$
$$D[\lambda A, \lambda B] = \lambda D[A,B], \tag{1.3}$$
$$D[A,B] \le D[A,C] + D[C,B], \tag{1.4}$$

for all $A, B, C \in K_c(R^n)$ and $\lambda \in R_+$.

Let $A, B \in K_c(R^n)$. The set $C \in K_c(R^n)$ satisfying $A = B + C$ is known as the Hukuhara difference of the sets A and B and is denoted by the symbol

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