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COMPUTATION OF AN OVER-APPROXIMATION OF THE BACKWARD REACHABLE SET USING SUBSYSTEM LEVEL SET FUNCTIONS

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Abstract. In this paper, we present a method to decompose the problem of computing the backward reachable set for a dynamic system in a space of a given dimension, into a set of computational problems involving level set functions, each defined in a lower dimensional (subsystem) space. This allows the potential for great reduction in computation time. The overall system is considered as an interconnection of either disjoint or overlapping subsystems. The projection of the backward reachable set into the subsystem spaces is over-approximated by a level set of the corresponding subsystem level set function. It is shown how this method can be applied to two-player differential games. Finally, results of the computation of polytopic over-approximations of the unsafe set for the two aircraft conflict resolution problem are presented.

Keywords. Reachable sets; interconnected systems; over-approximations.
AMS (MOS) subject classification: 70H20, 49N70, 49N75, 93A14, 93A15

1 Introduction

Computation of reachable sets for dynamic systems has an important application in the automatic verification of safety properties and synthesis of safe controllers for air traffic systems [12, 18]. The exact reachable set boundary is known to be the zero level set of the viscosity solution [2] of a Hamilton-Jacobi type of partial differential equation (PDE) [14]. To the best of our knowledge, Leitmann was the first to recognize the relationship between Bellman functions and the boundaries of the reachable sets [11]. In [22] it was shown how to approximate boundaries of reachable sets with an arbitrary accuracy using smooth functions. In the sequence of papers by Khrustalev [4, 5], locally Lipschitz functions are used to describe, arbitrarily accurate, under- and over-approximations of reachable sets. Finally, polytopic [19] and ellipsoidal approximations [7, 8, 9, 10] were developed to approximate reachable sets for linear systems (with and without perturbations).

The numerical solutions which provide convergent approximations of reachable sets for dynamic systems have computational complexity which is exponential in the continuous variable dimension [14, 13]. To overcome this problem, in [3] the authors use a polytopic approximation method [19], based on