

## INCLUSION PRINCIPLE FOR DISCRETE-TIME TIME-VARYING SYSTEMS

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**Abstract.** In this paper the concept of inclusion is applied to dynamic linear discrete-time time-varying dynamic systems. Starting from general definitions, which include restriction and aggregation as special cases, inclusion conditions are derived. It is proved that any inclusion can be considered as a specific composition of sequences of restriction/ aggregation pairs. The paper contains also a formulation of the contractibility conditions for implementation of time-varying state-feedback controllers.

**Keywords.** Inclusion principle, restriction, aggregation, discrete-time time-varying systems

### 1 Introduction

A considerable attention among researchers in the field of complex and large-scale systems has been paid to the *inclusion principle*, which has been found to provide a convenient framework for diverse approaches to both analysis and design. The basic idea, applied to linear time-invariant systems, has been presented in [16], starting from the fundamental geometric arguments [1,31]. It has been found that *restriction* and *aggregation* represent special cases of inclusion, having fundamental theoretical and practical importance. Numerous successful applications of the inclusion principle to control design, including decentralized overlapping control and reduced order design, have been reported, *e.g.* [13,16,22,23,26,27]. Considerable attention is paid to the practically important problem of controller *contractibility* (*expandability*), in the sense of ensuring contraction/expansion of controllers from one state space to another [2,3,9,10,11,12,23,28]. Applications of the inclusion concept to performance indices, closed-loop systems with dynamic controllers, stochastic systems and nonlinear systems have been described, for example, in [7,9,10,19,24]. However, the application of the inclusion principle to dynamic *time-varying systems* has remained insufficiently clarified. In [15], this problem has been considered for the first time from the point of view of time-invariant expansions/contractions applied to the state inclusion, in the context of restriction and aggregation. In [7] the focus is on the contractibility of dynamic controllers for discrete-time time-varying systems with time-invariant expansions/contractions. In [4,5] the attention is still paid to