Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **11 (2004)** 105-118

Optimal Control for Evolution Equations With Discontinuous Vector Fields

N.U.Ahmed

School of Information Technology and Engineering
and Department of Mathematics
University of Ottawa
Ottawa, Ontario K1N6N5

Abstract. In this paper we consider the question of existence of optimal (relaxed or measure valued) controls for a general class of semilinear evolution equations with discontinuous vector fields. This is a nonstandard system presenting nonstandard problems. The system is known to posses only generalized or measure valued solutions. We consider control problems of this system and prove the existence of solutions and optimal controls from the space of finitely additive measure valued functions.

Key Words: Semigroups of Operators, Semilinear Equations, Discontinous Vector Fields, Finitely Additive Measures, Measure solutions.

Subject Classification AMS(MOS) : 34GXX, 34H05, 34K05, 58D25, 49J27, 93C25

1 Introduction

Let E and F be two separable Banach spaces with E denoting the state space and F the control space. Let us consider the controlled evolution equation

$$\dot{x} = Ax + f(t, x, u), t \geq 0 \tag{1}$$

$$x(0) = \xi \tag{2}$$

in the Banach space E where A is the infinitesimal generator of a C_0-semigroup, $S(t), t \geq 0$, on E and $f : I \times E \times F \longrightarrow E$ is a measurable map.

Under sufficient regularity assumptions on the map f with strongly measurable F valued controls u, one can prove the existence of continuous solutions $x \in C(I, E)$. The control problem is to find a control policy over the time horizon I that minimizes the cost functional given by

$$J(u) \equiv C(u, x) \equiv \int_I \ell(t, x(t), u(t))dt + \Phi(x(T)), \tag{3}$$