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PERIODIC SOLUTIONS FOR A CLASS OF SINGULARLY PERTURBED SYSTEMS †

Mikhail Kamenski[1], Oleg Makarenkov[1] and Paolo Nistri[2]

[1]Department of Mathematics,
Voronezh State University, Voronezh, Russia.
[2] Dipartimento di Ingegneria dell'Informazione,
Università di Siena, 53100 Siena, Italy.

Abstract. In this paper we provide conditions to ensure the existence, for $\epsilon > 0$ sufficiently small, of periodic solutions of given period $T > 0$ for a class of singularly perturbed first order differential systems. Here $\epsilon > 0$ is the perturbation parameter. Our approach, based on the topological degree theory and the averaging theory, permits to weaken the conditions in ([5], Theorem 2) under which the existence of periodic solutions is proved.

Keywords: periodic solutions, averaging method, singular perturbations, topological degree.

AMS (MOS) subject classification: 34C25, 34C29, 34D15, 47H11.

Introduction

The starting point for the present work is the paper by K. Schneider [5], devoted to the extension of the theory of vibrational stabilizability to singularly perturbed first order control problems. A basic tool for such extension is represented by the averaging theory. In order to apply this theory the author assumes that an appropriate coordinate trasformation of the slow variable is a periodic diffeomorphism of fixed period. Indeed, this trasformation reduces the considered system to the standard form for the application of the classical averaging principle, (see, for instance, [2]); this, in turn, permits to prove the existence of periodic solutions for sufficiently small values of the perturbation parameter $\epsilon > 0$. The change of variable is introduced by means of a differential equation which is supposed to have a T-periodic solution for any initial condition in a suitable ball (assumption (A_3) of [5]).

Following [1], [4] and [6] we propose here an approach which combines the topological degree theory and the averaging theory. This approach allows us to assume the periodicity condition for the coordinate trasformation only on

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