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STABILITY OF STOCHASTIC EVOLUTION EQUATIONS IN HILBERT SPACES[1 1]

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Abstract. The objective of this article is to study exponentially stability of a class of stochastic evolution equations in Hilbert spaces. We give a condition which is equivalent to exponentially stability of C_0-semigroup of linear operators. By using this condition, we prove that weakly exponentially stability implies exponentially stability for a class of linear stochastic evolution equations.
Keywords. weakly exponentially stability, stochastic evolution equation, C_0-semigroup, Orlicz space, Young's function.
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1 Introduction

Let $S(t), t \geq 0$ be a strongly continuous semigroup of bounded operators on a Banach space H and A its infinitesimal generator. Then $S(t)y_0$ corresponds to the differential equation

$$dy/dt = Ay, \ y(0) = y_0. \tag{1}$$

Pazy [5] has shown the following:

Theorem 1.1. *The two statements below are equivalent:*
(I) $\int_0^\infty |S(t)y|^p dt < \infty, y \in H$ *for some* $p \geq 1$.
(II) $|S(t)| \leq M \exp\{-\alpha t\}, t \geq 0$ *for some* $M \geq 1$ *and* $\alpha > 0$.

In the above $|\cdot|$ denotes the norm of vectors or operators. Zabczyk [16] weakened the condition **(I)** by replacing $|S(t)y|^p$ by $m(|S(t)y|)$, where m is a strictly increasing convex function with $m(0) = 0$. This condition is also equivalent to the stronger one:
(I') $\int_0^\infty |S(t)y|^p dt \leq K|y|^p, y \in H$ for some $p \geq 1$ and $K > 0$.

Pritchard and Zabczyk posed the following open problem in [8]. Does the condition:
(III) $\int_0^\infty |(S(t)x, y)|^p dt < +\infty$, for some $p \geq 1$ and all $x, y \in H$

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