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Integral Average Method For Oscillation of Matrix Differential Systems With Damping [1]

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Abstract. Using a generalized Riccati transformation, some new oscillation criteria for linear second order matrix differential system with damping are found through the method of integral average. These results can be considered as generalizations and improvements of the results due to Kamenev and Yan in scalar cases.
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1 Introduction

In this paper, we consider oscillatory properties for the linear second order matrix differential system with damping

$$Y'' + R(t)Y' + Q(t)Y = 0, \quad t \in [t_0, \infty), \tag{1}$$

where $R(t), Q(t)$, and $Y(t)$ are $n \times n$ continuous matrix-valued functions, $R(t)$ and $Q(t)$ are symmetric. When $R(t) \equiv 0$, system (1) reduces to the linear second order matrix differential system

$$Y'' + Q(t)Y = 0, \quad t \in [t_0, \infty). \tag{2}$$

By M^* we mean the transpose of the matrix M, for any $n \times n$ symmetric matrix M, it eigenvalues are real numbers, we always denote by $\lambda_1[M] \geq \lambda_2[M] \geq \cdots \geq \lambda_n[M]$. A solution $Y(t)$ of (1) (or (2)) is said to be nontrivial solution if $\det Y(t) \neq 0$ for at least one $t \in [t_0, \infty)$, and a nontrivial solution $Y(t)$ of (1) (or (2)) is said to be prepared if

$$\begin{aligned} &Y^*(t)Y'(t) - (Y^*(t))'Y(t) \equiv 0, \\ &Y^*(t)R(t)Y'(t) - (Y^*(t))'R(t)Y(t) \equiv 0, \quad t \in [t_0, \infty). \end{aligned}$$

System (1) (or (2)) is said to be oscillatory on $[t_0, \infty)$ in case the determinant of every nontrivial prepared solution vanishes at least one point on $[T, \infty)$ for each $T \geq t_0$.

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