

A NOTE ON QUASILINEARIZATION FOR IMPULSIVE SYSTEMS

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Abstract. A method of quasilinearization coupled with the method of lower and upper solutions is applied to an initial value problem for a system of nonlinear impulsive differential equations. The nonlinear terms in the differential equation and in the impulse are decomposed according to monotonicity properties. The definitions of upper and lower solutions are modified to accommodate these decompositions. An algorithm to construct monotone sequences of successive approximation to the solution is given. Uniqueness of solutions guarantees that limits of the sequences do give the solution of the original problem; it is proved that the convergence is quadratic.

Keywords. impulsive differential equations, mixed couple of lower and upper solutions, quasilinearization, quadratic convergence.

AMS (MOS) subject classification: 34A37, 34E05.

1 Introduction

The theory of impulsive differential equations is a relatively new branch in the field of differential equations. We refer to the monograph [2] for background material with respect to impulsive equations. Impulsive equations have been useful in many applications in science and technology; hence, the development of the theory has been rapid.

The method of quasilinearization is a practical method to obtain approximate solutions of nonlinear problems. The origin of this method lies in the theory of dynamic programming [4]. Lakshmikantham, Leela and various co-authors ([7], [9], [10], [12]) have generalized and extended to method to apply to a wide variety of problems. See the authoritative recent monograph [11] for a good background in the methods and applications of quasilinearization. The quasilinearization method is a Taylor series numerical method in which the truncation is chosen so that the convergence of the iterates is quadratic. Methods of rapid convergence, generalizations of quasilinearization methods have been recently established [5]; the truncation is chosen so that the rate of convergence is of higher order.

Quasilinearization methods have been employed on impulsive equations. An introduction for initial value problems is given in the monograph [11].