Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **11 (2004)** 81-100

REGULATING DISCRETE-TIME STOCHASTIC SYSTEMS: FOCUSING ON THE PROBABILITY DENSITY FUNCTION

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Abstract. With motivations from the processing and manufacturing industries, the relationship between dynamics and stationary probability density function (PDF) is investigated for discrete-time stochastic nonlinear system models. Since this relationship is given by an integral equation for which no general and few specific solutions are known, an approximate solution is developed. By parameterizing both the system dynamics and the PDF, the integral equation is reduced to a set of algebraic equations in the two parameter sets. By choosing values for either parameter set, the other set of values can be obtained. The proposed approach is applied to two controller synthesis problems.
Keywords. non-Gaussian processes, nonlinear systems, feedback control methods, stationarity, stochastic control, discrete-time systems
AMS (MOS) subject classification:

1 Introduction

In the processing and manufacturing industries, systems are subject to unmeasurable, random disturbances such as non-uniform feedstocks, sensor inaccuracies, fluctuating ambient conditions, etc. The standard approach to systems analysis and controller synthesis in such cases relies on linear system models, objectives expressed as the expectation of a quadratic function, and disturbances with Gaussian distributions. This approach is used in the vast majority of results published in the process control literature. The important advantage of this linear-quadratic-Gaussian (LQG) framework is that theoretical developments can be made without explicit reference to the probability density function (PDF) of the system variables. Unfortunately, in industrial systems, any of the three tenets of the LQG framework do not hold, leading to inaccurate models, poor process performance, controller instabilities, and so forth. Any framework that recognizes the practical realities of industrial systems must include nonlinear system models, disturbances with non-Gaussian PDFs and nonquadratic performance metrics. The major barriers (*i.e.*, lack