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NUMERICAL PROCEDURES FOR RECOVERING A TIME DEPENDENT COEFFICIENT IN A PARABOLIC DIFFERENTIAL EQUATION

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Abstract. In this paper we study a finite difference approximation to an inverse problem of finding the function $u(x,t)$ and the unknown positive coefficient $a(t)$ in a parabolic initial–boundary value problem. The backward Euler scheme is studied and its convergence is proved via the application of the discrete maximum principle. Error estimates for u and a, and some experimental numerical results using the newly proposed numerical procedure are presented.

Keywords. Inverse problem, convergence, maximum principle, numerical method.
AMS (MOS) subject classification: Primary 35R30; Secondary 65M32, 76R50, 65M06.

1 Introduction

This paper discusses the problem of finding the function $u(x,t)$ and the unknown positive coefficient $a(t)$ in the parabolic initial–boundary value problem

$$\begin{aligned} u_t &= a(t)u_{xx}, && \text{in} \quad Q_T, \\ u(x,0) &= \phi(x), && 0 \le x \le 1, \\ u(0,t) &= g_1(t), && 0 \le t \le T, \\ u(1,t) &= g_2(t), && 0 \le t \le T, \end{aligned} \tag{1.1}$$

where $Q_T = \{(x,t) : 0 < x < 1, 0 < t < T\}$, $T > 0$, and ϕ, g_1, g_2 are known functions.

With only the above data this problem is under–determined and we are forced to impose an additional boundary condition, such that a unique solution pair (u,a) is obtained. In particular, this may take the form of the heat flux $h(t)$ at a given point $x^* = 0$ or 1, that is,

$$-a(t)u_x(x^*,t) = h(t), \qquad 0 \le t \le T. \tag{1.2}$$