

CONTROLLING HOPF BIFURCATION OF NONLINEAR DYNAMICAL SYSTEMS

Changpin Li

Department of Mathematics

Shanghai University, Shanghai 200436, P. R. China

Abstract. Hopf bifurcation has been a classical subject for study, yet the investigation of its control is relatively new. This paper discuss a general methodology of controlling Hopf bifurcation emerged in nonlinear dynamical systems. For notational simplicity, this topic is addressed under the three-dimensional setting, but all analysis and results can be theoretically extended to n -dimensional systems.

Keywords. nonlinear dynamical system, Hopf bifurcation, bifurcation control, equilibrium, the first Lyapunov coefficient.

AMS (MOS) subject classification: 34B15

1 Introduction

Controlling bifurcations have been increasing interest recently due to its potential applications in applied sciences and technology ([1-5], [9] and references therein). Among all controlling bifurcations, controlling Hopf bifurcation is especially important since Hopf bifurcation provides a transition from local static dynamics to global ones and we can modify some dynamical characteristics of a parameterized systems via controlling their Hopf bifurcation. Although there exists a large number of publications, in both theoretical analysis and numerical calculations as well as in real applications, on Hopf bifurcation studies ([6], [10] and [11] and cited quantities of publications therein). However, the new subject of Hopf bifurcation control is relatively less known and, as a matter of fact, its investigation has seen very little progress to date ([1], [4], etc.). In this paper, more effort is devoted to the investigation of a general methodology for the Hopf bifurcation control problem.

Consider the following general nonlinear dynamical system:

$$\dot{x} = f(x, \alpha), \quad x \in R^n, \quad (1)$$

in which $\alpha \in R$ is a variable system parameter. Suppose that x_0 is an equilibrium of system (1), i.e.,

$$f(x_0, \alpha) = 0. \quad (2)$$