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# SYNCHRONIZATION IN A NETWORK OF WEAKLY COUPLED OSCILLATORS WITH PHASE-PLUS-FREQUENCY ADAPTION

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**Abstract.** Synchronization in a connected network of weakly coupled oscillators is investigated. It is proved that if each oscillator adjusts its phase and frequency simultaneously according to the error between the phase of the oscillator and the mean of the phases of other oscillators connected to this oscillator, then all the oscillators in the network will synchronize asymptotically, no matter what coupling structure the network has and no matter how small the coupling strength of the network is.
**Keywords.** Synchronization; Network; Oscillators; Stability; Dynamics.

## 1 Introduction

Synchronization of coupled oscillators is a ubiquitous phenomenon in science, nature, engineering and social life [1, 2]. Systems as diverse as clocks, singing crickets, cardiac pacemakers, firing neurons, fireflies and applauding audiences exhibit a tendency to operate in synchrony. These phenomena are universal and can be understood within a common framework based on nonlinear dynamics. One of the earlier seminal work in this area is due to Winfree [3]. He simplified the problem by assuming that the oscillators are strongly attracted to their limit cycles, so that amplitude variations can be neglected and only phase variations need to be considered. Later, Kuramoto put Winfree's intuition about phase models on a firmer foundation [4]. He showed that for any system of weakly coupled, nearly identical limit-cycle oscillators, the long-term dynamics are given by phase equations of a simple universal form.

The original Kuramoto model is a globally coupled (mean-field) model, where each oscillator is coupled directly to all others and so is coupled to the mean of all the individual influences. Although globally coupled model captures some important properties of real networks, it is easy to notice its limitations: a globally coupled network with $N$ nodes has $N(N-1)/2$ links, while most large-scale real networks tend to be sparse, that is, the number of edges in a real network is generally of order $N$ rather than $N^2$. In recent years, network of Kuramoto oscillators with nearest-neighbor, random or small-world connections has been investigated [5]. A fundamental result