

## Chaotifying Continuous-Time Takagi-Sugeno Fuzzy Systems via Time-Delay Feedback

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**Abstract.** An approach based on time-delay feedback control is proposed to make a given stable continuous-time Takagi-Sugeno (TS) fuzzy system chaotic. It comprises fuzzy feedback linearization and a suitable approximate relationship between a time-delay differential equation and a discrete map. The time-delay feedback controller, chosen among several candidates, is a simple sinusoidal function of the system's delay states, which has small amplitude. This approach is mathematically proven correct for rigorous generation of chaos from stable continuous-time TS fuzzy systems, where the generated chaos is in the sense of Li and Yorke. Numerical examples are included to visualize the theoretical analysis and the controller design.

**Keywords.** Chaotification; continuous-time TS fuzzy systems; fuzzy-model-based controller (FMBC); time-delay feedback; fuzzy feedback linearization

**AMS (MOS) subject classification:**

## 1 Introduction

In contrast to the main stream of ordering or suppressing chaos, the task of making a nonchaotic dynamical system chaotic, or retaining the existing chaos of a chaotic system, known as “chaotification” or “anticontrol of chaos,” has attracted more and more attention from the engineering and physics communities in recent years [1–8]. There are many practical reasons for chaotification. For instance, chaos has impacts on some novel time- and/or energy-critical applications. Also, applications have been found in designing high-performance circuits and devices (e.g., delta-sigma modulators and power converters), in liquid mixing, chemical reactions, biological systems (e.g., in the human brain, heart, and perceptual processes), secure communication and information processing, and in critical decision-making [1]. Some very desirable features of chaos in these applications have provided control system designers with new ways of solving many nontraditional problems.

Interactions between fuzzy logic and chaos theory have been explored since 1990s and increasing attention has been received in this area [9–13].