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CONTROLLING BIFURCATIONS IN MAPS VIA A FREQUENCY-DOMAIN APPROACH

María Belén D'Amico, Jorge L. Moiola and Eduardo E. Paolini

Departamento de Ingeniería Eléctrica y de Computadoras
Universidad Nacional del Sur
Avda. Alem 1253, B8000CPB, Bahía Blanca, Argentina
mbdamico@criba.edu.ar

Abstract. A unified frequency-domain methodology to analyze Hopf and period doubling bifurcations in maps is presented. The technique relies on the harmonic balance method and feedback systems theory. The results are applied for controlling the bifurcation structure of the Hénon map. A description of the different regions on the two-parameter plane summarizes the diverse dynamical behaviors.

Keywords: bifurcation control, discrete-time systems, frequency-domain analysis, harmonic balance method.
AMS (MOS) subject classification: 93B52; 93C55; 93CC80; 37C25; 37G10.

1 Introduction

The dynamical behavior of maps is generally unexpected and complex. Phenomena such as periodic orbits, quasi-periodic orbits or even chaotic motion can be commonly observed. The most famous and classical scenario is the so-called period doubling route to chaos in which a sequence of consecutive period doublings (fixed points of period-2^q, where q is an integer number) is generated through the variation of one distinguished parameter. Another complex case consists in the creation of an invariant cycle from a fixed point, generally after loosing its stability by varying one parameter. This phenomenon is known as Hopf bifurcation, and the emerging invariant orbit can also be transformed into a chaotic attractor for parameter values away from the critical condition.

Period doubling and Hopf bifurcations have been studied extensively due to their important implications not only in maps but also in flows via the so-called return map or Poincaré section. The Hopf bifurcation for maps has a rich history paralleling the development of the rather similar Hopf bifurcation for flows. However, the first one is more complex than its analogue counterpart as it has been well documented in [2], [10] and [17]. Furthermore, the complexity of the dynamics in the vicinity of Hopf bifurcation curves is truly remarkable. For instance, close to a Hopf bifurcation curve in the two-parameter plane there may exist horn-shaped resonance regions which