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SINGULAR PROBLEMS ARISING IN CIRCULAR MEMBRANE THEORY

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Abstract. An existence result is presented for a singular second order boundary value problem arising in circular membrane theory.

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1 Introduction

The equation for a circular membrane (subjected to a normal uniform pressure) can be reduced [3] to

$$y'' + \frac{k}{y^2} + \frac{3}{x}\,y' = 0,\ 0 < x < 1; \tag{1.1}$$

here $k > 0$ is a constant, x is the radial coordinate and $y(x)$ the radial stress. At the edge $(x = 1)$ we have the condition

$$a_0\,y(1) + y'(1) = 0,\ a_0 > 0 \quad \text{or} \quad y(1) = \lambda > 0 \tag{1.2}$$

and at the center (for symmetry)

$$y'(0) = 0. \tag{1.3}$$

The boundary value problem (1.1)–(1.3) was discussed in [5], and the idea involved approximating the singular problem by a sequence of nonsingular problems and then using a limiting argument. However we will show that these results can be deduced immediately from the upper and lower solution theory for singular problems presented by Bobisud and O'Regan [1] in 1994. In Section 2 we first present the theory from [1]. The proof is included since it is short and straightforward. A discussion of (1.1)–(1.3) will then conclude the section. Our argument is elementary, so there is no need for the "approximating analysis" or the "infinite interval analysis" presented in [2, 5].