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EXISTENCE AND ORDER-STABILITY OF EQUILIBRIUM SOLUTIONS FOR HIGHER ORDER PARABOLIC EQUATIONS

Bendong Lou

Graduate School of Mathematical Sciences, University of Tokyo,
Komaba 3-8-1, Meguro-ku, Tokyo 153, Japan

Abstract. We study the equilibrium solutions for 1-dimensional higher order parabolic equations in the presence of sub- and supersolutions which are well-ordered: "subsolution $<$ supersolution",or non-well-ordered: "subsolution $\not\leq$ supersolution". The existence, order-stability (order-instability) for the extremal equilibrium solutions are obtained. For second order equations, the order-stability (order-instability) defined in this paper implies general stability (instability).
Keywords. Higher order parabolic equations, equilibrium solutions, existence and order-stability, subsolutions, supersolutions.
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1 Introduction

Consider the initial-boundary value problem (IBVP) for a 1-dimensional higher order semilinear parabolic equations:

$$(\widetilde{E}) \qquad \begin{cases} u_t = (-1)^{m+1} \dfrac{\partial^n u}{\partial x^n} + f_1(x,u), & (0 < x < 1, \quad t > 0), \\ \dfrac{\partial^i u(0,t)}{\partial x^i} = 0, \quad i = 0,1,2,\cdots,n-2, & u(1,t) = 0, \qquad (t > 0), \\ u(x,0) = \tilde{u}(x), & (0 < x < 1). \end{cases}$$

where $m > 0$ is a fixed integer, $n = 2m$, $f_1 \in C^1[J \times \mathbb{R}, \mathbb{R}]$ $(J = [0,1])$, $\tilde{u}$ is in a suitable space which will
be given below. A function $v(x)$ is called an *equilibrium solution* of $(\widetilde{E})$ if it solves

$$(E) \qquad \begin{cases} (-1)^m v^{(n)} = f_1(x,v), & (0 < x < 1), \\ v(0) = v'(0) = \cdots = v^{(n-2)}(0) = 0, & v(1) = 0. \end{cases}$$

When $m = 1$, that is, the equations are the second order ones, problems $(\widetilde{E})$ and (E), as well as their analogues in higher dimensional spaces have been investigated a lot. Among others, the study in the presence of sub- and supersolutions has been improved largely since 1970's (see e.g. [1,6,7,12,14,15,17],etc.)