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Stability in Terms of Two Measures for Stochastic Differential Equations

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Abstract. There are many different types of stability concepts for stochastic differential equations. A key question is whether a general stability theory can be derived which unifies a variety of known concepts of stability in a single set up. The main aim of this paper is to give a positive answer to this question by introducing the concept of stability in terms of two measures for stochastic differential equations. We shall employ the method of Lyapunov functions to establish a number of sufficient criteria for the stability in terms of two measures. Our new stability theory can not only be used to show a given system to be stochastically stable in the classical sense, but can also be used to deal with some situations where the classical stability theory is not applicable.
Keywords. Stochastic stability, two measures, definite, descrescent, Itô formula.

1 Introduction

Consider the ordinary differential equation

$$\dot{x}(t) = f(x(t), t), \quad t \geq t_0 (\geq 0), x(t_0) = x_0, \tag{1}$$

for which a unique solution is assumed to exist. The original idea of Lyapunov stability for the solution of Eq. (1) has given rise to many new concepts that are important in the theory of ordinary differential equation. These include: eventual stability, partial stability, relative stability, conditional stability, total stability, corresponding boundedness and practical notions. A significant amount of literature exists for each of these concepts. It is natural to ask, whether we can develop a theory which unifies and includes a variety of known concepts of stability in a single set up? The answer is yes and it involves the development of stability in terms of two measures. This has been investigated for Eq. (1) by many authors, for instance, Dos Reis and Benά [2], Lakshmikantham and Xinzhi Liu [6, 7, 8] and Movchan [11].

Often, due to various uncertainties in the environment and within the system, Eq. (1) is perturbed and modeled as an Itô stochastic differential equation

$$dx(t) = f(x(t), t)dt + g(x(t), t)dB(t) \tag{2}$$