Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 875-893

REAL DYNAMICS OF A 3-POWER EXTENSION OF THE $3x + 1$ FUNCTION

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Abstract. This paper investigates the real dynamics of a generalization of the $3x + 1$ function using powers of three. We will see that any cycle of positive integers is attractive for this generalization and the cycle has an expansion factor given by Terras' coefficient function. We will see the function has a negative Schwarzian derivative for $x \geq 0$ and will be able to identify invariant intervals and approximately locate the fixed points and critical points. The special simplicity of dynamics around the cycle $(1, 2)$ means there is a natural generalization of total stopping time for this function. We conjecture that the odd critical points of this generalization are well behaved. In particular, they lie in the immediate basin of total stopping time surrounding each odd integer.

Keywords. Collatz Problem, Syracuse Problem, $3n + 1$, Hailstone Function, Real Dynamics

AMS (MOS) subject classification: 26A18, 11B83, 37C25

1 Introduction

The classic $3x + 1$ problem concerns the iteration of a function that results in $3x + 1$ for odd x and $x/2$ for even x. The classic $3x + 1$ conjecture is that iteration of that function upon a positive integer eventually reaches the cycle containing 1. However, there are slight variations on the function and there are many related conjectures now in the literature. Lagarias gives an overview of important early results regarding the $3x + 1$ problem in [5] and maintains an annotated bibliography of the subject [7]. The literature of the subject is rapidly growing and includes Wirsching's book [14], which is another source that provides an overview of the literature. Considerable amounts of information on the $3x + 1$ problem appear on the web; good starting points include the web version of [5] and Roosendall's site which includes search results [11].

Notice that when x is odd, then $3x+1$ is even, and hence the next iteration of the classic function will be division by 2. Thus, many of the results known about the $3x + 1$ problem may be easily described in terms of the following