

## A BOUNDARY VALUE PROBLEM FOR SECOND ORDER IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS IN A BANACH SPACE

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**Abstract.** By establishing a comparison result and using the upper and lower solutions, the authors obtain an existence theorem of the maximal and minimal solutions for a boundary value problem of second order nonlinear impulsive integro-differential equations of mixed type on an infinite interval with an infinite number of impulsive times in a Banach space.

**AMS(MOS) subject classifications:** 34A37, 34C25, 45J05

### 1 Introduction

This is a continuation of article [1]. In article [1], we have discussed the maximal and minimal solutions of a boundary value problem for first order nonlinear impulsive integro-differential equations of mixed type on an infinite interval with an infinite number of impulsive times in a real Banach space by means of establishing a comparison result and using the upper and lower solutions. In this paper, we shall investigate a boundary value problem for second order such equations. In this situation, the basic space  $BPC[J, E]$  (i.e. the space of bounded piecewise continuous functions) in [1] is not applicable and we must introduce a new space  $DPC^1[J, E]$  which deals with a class of unbounded piecewise continuously differentiable functions. On the other hand, the comparison result is more difficult and complicate. This paper is organized as follows. In Section 2, we establish a comparison result and prove several lemmas. By using the method of upper and lower solutions, we then obtain, in Section 3, an existence theorem for the maximal and minimal solutions of the boundary value problem. An example is also worked through to illustrate our result. The problem formulation is given in the rest of this section.

Consider the boundary value problem (BVP) for second order nonlinear impulsive integro-differential equation of mixed type in a real Banach space

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