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DIFFERENTIAL EQUATIONS IN METRIC SPACES: AN INTRODUCTION AND AN APPLICATION TO FUZZY DIFFERENTIAL EQUATIONS

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Abstract. We introduce the concept of differential equation in a metric space and apply it to the study of an initial value problem for a fuzzy differential equation.
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1 Introduction

Consider a differential equation in a given space X

$$u'(t) = f(t, u(t)) \, , \, t \in [0, T] \tag{1}$$

with a specified initial condition

$$u(0) = u_0 \, , \tag{2}$$

where $T > 0$, and $f : [0, T] \times X \to X$.

If $X = \mathbf{R^n}$, the classical Peano's theorem gives us the existence of a (local) solution $u : [0, \delta] \to \mathbf{R^n}$ of the initial problem (1)-(2) for some $\delta \in (0, T]$. If the function f is, for example, linear in u or bounded, then we can take $\delta = T$ and have a global solution on the interval $[0, T]$. Moreover, if f satisfies a Lipschitz condition in the second variable u, then the Picard-Lipschitz theorem ensures the existence of a unique solution.

In the case that X is a Banach space, the continuity of f does not guarantee the existence of a local solution, i.e., Peano's theorem does not hold, but the Picard-Lipschitz theorem is still valid.

To deal with fuzzy differential equations, we have to consider the space $X = E^n$. Interpreting the derivative in the sense of Hukuhara, it is known the validity of the existence and uniqueness Picard-Lipschitz theorem [10]. Peanos's theorem is still valid under some additional conditions [15,17,18].