Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **10 (2003)** 755-764

A HIGHLY ACCURATE DERIVATIVE RECOVERY FORMULA FOR FINITE ELEMENT APPROXIMATIONS IN ONE SPACE DIMENSION

Tie Zhang[1], Changjun Li[2], Yiyong Nie[3] and Ming Rao[4]

[1,2,3]Shenyang Institute of Automation, Academia Sinica, Shenyang 110016
[1,2]Department of Mathematics, Northeastern University, Shenyang 110006
[4]Department of Chemical Engineering, University of Alberta
Edmonton, Alberta T6G 2G6

Abstract. A highly accurate derivative recovery formula is presented for the k-order finite element approximations to the two-point boundary value problems. This formula possesses the superconvergence of $O(h^{k+1})$ order on the whole domain in L_∞ norm and the ultraconvergence of $O(h^{2k})$ order at the mesh points, and also the lowest regularity requirement for the exact solutions. Numerical experiments are given to verify the high accuracy of our formula.

Keywords. finite element, derivative recovery, ultraconvergence.
AMS (MOS) subject classification:76S05, 45K05, 65M12, 65M60, 65R20

1 Introduction

It is well known that for the finite elements of Lagrange type we can not directly calculate the derivative values at the mesh points because of the discontinuity of the derivative functions of finite element solutions. Traditionally, the difference quotient methods are used to produce the derivative values by means of the function values, but the approximation accuracy is not good enough. In recent years, many derivative recovery techniques are presented in order to obtain the approximations of finite element derivatives. For examples, the averaging technique, the Z-Z patch recovery technique and the patch interpolating recovery technique, etc., see, for example, [2-3, 5-11]. Usually, these methods possess superconvergence of one order higher than the optimal convergence rates of finite element approximations in derivative and require the exact solutions to be in W^3_∞ at least.

In this article, we will establish a new highly accurate derivative recovery formula for the k-order finite element approximations to the two-point boundary value problems. It will be seen that our formula has the following global superconvergence:

$$\| u' - Ru'_h\|_{0,\infty} \le Ch^{k+1}\|u\|_{k+1,p},\ p > 1,\ k \ge 1, \tag{1}$$