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MODELLING OF THE MOTION OF A DISK ROLLING ON A SURFACE OF REVOLUTION

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Abstract. This work deals with the modelling of the motion of a disk rolling without slipping on a rigid terrain. This terrain is described as a surface of revolution as given by (1). It is assumed here that the motion of the disk is controlled by a tilting moment, a directional moment and a pedalling moment. A framework for the mathematical model, describing the motion of the disk, is derived.
Keywords. Rolling disk, surface of revolution, rigid surface, nonholonomic constraints.
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1 Introduction

This work deals with the modelling of the motion of a disk rolling without slipping on a (rigid) surface of revolution. It is assumed here that the motion of the disk is controlled by a tilting moment, a directional moment and a pedaling moment.[1] The description of such a motion demonstrates a simple case of interaction between the motion of a rigid body and the environment. The results obtained here can be extended to the modelling and control of the motion of vehicles on a rigid terrain. This work is to some extent a continuation of [8], where the problem of modelling and control of the motion of a disk, rolling on a rigid terrain, in which the changes in the terrain take place along one direction, is dealt with. For the control and guidance of the motion of a disk rolling on a horizontal plane see for example [2,6,7] and the references cited there.

In this work, the fundamental building blocks for the dynamical model of the motion of the disk, are constructed for a class of surfaces, as given by (1). That is, the mathematical expressions for the disk's angular velocity vector, the disk's center of mass velocity vector, the disk's potential energy, and the nonholonomic constraints at the point of contact between the disk and the surface, are derived for a class of surfaces. Using these results, one can construct, for a given member of this class of surfaces, the Lagrangian function [5], and consequently, derive the Lagrange equations for the disk's motion.

In addition, the above mentioned results are implemented here in an example

[1] For the generation of such moments see for example [2].