

OPTIMAL FILTERING AND CONTROL FOR THIRD DEGREE POLYNOMIAL SYSTEM

Michael V. Basin and Maria A. Alcorta Garcia

Department of Physical and Mathematical Sciences
Autonomous University of Nuevo Leon
Apdo postal 144-F, C.P. 66450, San Nicolas de los Garza
Nuevo Leon, Mexico

Abstract. This paper presents the optimal nonlinear filter for a stochastic system state given by a polynomial equation of degree 3 or 4 and linear observations confused with white Gaussian noises. The obtained filtering equations are applied to solution of the state estimation problem for a nonlinear automotive system. Simulation results are compared for the optimal polynomial filter given in this paper and the linear Kalman-Bucy filter applied to the linearized system. Using the duality principle, the optimal regulator is then designed for a polynomial system of degree 3 with linear control input and quadratic cost criterion, applied to the nonlinear automotive system, and compared to the optimal linear regulator dual to the Kalman-Bucy filter.

Keywords. polynomial system, optimal filtering, optimal control, automotive application.

AMS (MOS) subject classification: 93E11, 60G35, 49K15.

1 Introduction

Although the general optimal solution of the filtering problem for nonlinear state and observation equations confused with white Gaussian noises is given by the Kushner equation for the conditional density of an unobservable state with respect to observations [6], there are a very few known examples of nonlinear systems where the Kushner equation can be reduced to a finite-dimensional closed system of filtering equations for a certain number of lower conditional moments. The most famous result, the Kalman-Bucy filter [5], is related to the case of linear state and observation equations, where only two moments, the estimate itself and its variance, form a closed system of filtering equations. However, the optimal nonlinear finite-dimensional filter can be obtained in some other cases, if, for example, the state vector can take only a finite number of admissible states [13] or if the observation equation is linear and the drift term in the state equation satisfies the Riccati equation $df/dx + f^2 = x^2$ (see [3]). The complete classification of the "general situation" cases (this means that there are no special assumptions on the structure of state and observation equations), where the optimal nonlinear finite-dimensional filter exists, is given in [14].