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# A CONSTRAINT OPTIMIZATION PROBLEM IN SINGULAR CONTROL THEORY: WHERE IS THE NEAREST NON-INDEX-AT-MOST-ONE MATRIX PENCIL?

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**Abstract.** In this paper the distance from a regular matrix pencil of index at most one to the pencils which are not regular or are regular but of index higher than one is considered. The algebraic characterizations of such distance with structured and unstructured perturbations are obtained as constraint optimization problems. The characterizations lead to some important bounds for the relevant distance.
**Keywords.** Matrix pencil, regularity, index, perturbation, distance, optimization.

## 1 Introduction

Let $E, A \in \mathbf{R}^{n\times n}$. $(E, A)$ is regular if and only if $\det(\alpha E - \beta A) \neq 0$ for some $(\alpha, \beta) \in \mathbf{C}^2$. It is well-known that for a regular pencil $(E, A)$, there exist nonsingular matrices $M$ and $N$ such that

$$M(sE - A)N = \begin{bmatrix} sI - L & \\ & sJ - I \end{bmatrix},$$

where the eigenvalues of $L$ coincide with the finite eigenvalues of the pencil and $J$ is a nilpoten Jordan matrix such that $J^i = 0, J^{i-1} \neq 0$, for some $i > 0$, corresponding to the infinite eigenvalues. The index of the system, denoted by $\mathrm{ind}(E, A)$, is defined to be the degree $i$ of nilpotency.

This paper studies the norm-wise distance from a regular pencil of index at most one to the nearest non-regular pencil or regular pencil but with index higher than one. Our work is motivated by control theory of singular systems and [7].

Consider a singular system of the form

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t), \tag{1}$$

where $E, A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times m}, C \in \mathbf{R}^{p\times n}$ and $E$ is singular. Existence and uniqueness of (classical) solutions to (1) are guaranteed if $(E, A)$ is regular [1]. For systems that are regular and of index at most one, they can be separated