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THE IMMERSED INTERFACE METHOD FOR ELASTICITY PROBLEMS WITH INTERFACES

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Dedicated to Professor John R. Cannon on the occasion of his 65-th birthday.

Abstract. An immersed interface method for solving linear elasticity problems with two phases separated by an interface has been developed in this paper. For the problem of interest, the underlying elasticity modulus is a constant in each phase but vary from phase to phase. The basic goal here is to design an efficient numerical method using a fixed Cartesian grid. The application of such a method to problems with moving interfaces driving by stresses has a great advantage: no re-meshing is needed. A local optimization strategy is employed to determine the finite difference equations at grid points near or on the interface. The bi-conjugate gradient method and the GMRES with preconditioning are both implemented to solve the resulting linear systems of equations and compared. Numerical results are presented to show that the method is second-order accurate.

Keywords: elasticity, interfaces, jump conditions, finite differences, the immersed interface method.

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1 Introduction

Elasticity problems of multiple phase elastic materials separated by phase interfaces often arise in materials science [18, 21]. Two important examples of such problems occur in the microstructural evolution of precipitates in an elastic matrix due to the diffusion of concentration and in the morphological instability due to stress-driven surface diffusion in solid thin films, cf. e.g., [2, 10, 13] and the references therein. The understanding of these physical processes is crucial to improve material stability properties, and in turn to develop new and advanced materials that have applications in automobile manufacture, aircraft industries, and modern communication technologies. However, solving such elasticity problems are often very difficult due to complicated geometries, multiple components, and nonlinearities that appear in these problems. For these reasons, there has been a great interest recently, in all materials science, scientific computing, and applied mathematics communities, in developing efficient and accurate numerical techniques for elasticity problems with interfaces separating multiple phases.