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A NONLINEAR STABILITY ANALYSIS OF PATTERN FORMATION IN ISOTHERMAL THIN LIQUID FILMS

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Abstract. The development of equilibrium patterns in isothermal thin liquid films is investigated by means of a hexagonal-planform weakly nonlinear stability analysis applied to the appropriate lubrication equation describing layer thickness. The main result of this analysis is that supercritical equilibrium patterns can occur for an interval of mean layer thickness with subcritical rupture occurring outside that interval. Those morphological phase separation patterns generated by the coupling between the long-range attractive and short-range repulsive intermolecular forces consist of surface ridges and hexagonal network-like cells or close-packed configurations of nanodroplets separated by relatively flat ultra thin films with cells being stable for the thicker layers; nanodroplets, for the thinner ones; and ridges, for layers of intermediate thickness. These predictions are in accord with both relevant experimental evidence and numerical simulations as well as being consistent with rupture occurring by hole formation in relatively thick layers but by drop formation in thinner ones.

Key words. isothermal thin liquid films, lubrication equations, nonlinear stability analysis, hexagonal pattern formation, morphological phase separation.

1 Introduction and Formulation of the Problem

Rayleigh-Bénard buoyancy-driven convection has to date provided perhaps the best studied example of nonlinear pattern selection (reviewed by Kosch - mieder [10]). One of the methods traditionally used to predict such pattern selection is a weakly nonlinear stability analysis that, although incorporating the nonlinearities of the relevant model system, basically pivots a perturbation procedure about the critical point of linear stability theory (reviewed by Wollkind *et al.* [32]). The advantage of such an approach over strictly numerical procedures is that it allows one to deduce quantitative relationships between system parameters and stable patterns which are valuable for experimental design and difficult to accomplish using simulation alone. Recently, there has been considerable interest generated in pattern formation and selection during the controlled plane-front solidification of a dilute binary alloy under the influence of an imposed temperature gradient and during