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MATHEMATICAL MODELING OF AXONEME MECHANICS AND FLUID DYNAMICS IN CILIARY AND SPERM MOTILITY

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Abstract. We present a fluid-mechanical model of an individual cilium or flagellum, which incorporates discrete representations of the dynein arms, and the passive elastic structure of the axoneme including the microtubules and nexin links. This model, based upon Peskin's immersed boundary method, couples the internal force generation of the molecular motors through the passive elastic structure with the external fluid mechanics governed by the Navier-Stokes equations. The flagellar and ciliary beats are not preset, but are an emergent property of the interacting components of the coupled fluid-axoneme system. The ciliary and flagellar waveforms are controlled by curvature-control mechanisms. We present numerical simulations of a ciliary beat and spermatozoa swimming.
Keywords. Immersed boundary method, fluid-structure interaction, axoneme, dynein, cilia, flagella

1 Introduction

The eukaryotic flagella and cilia are cellular organelles that propel cells through an aqueous environment or cause fluid or mucous flow over cells. They are chemomechanical in that they convert the chemical energy of ATP into movement. They do this by the activity of dynein ATPases that produce shear forces along microtubules that form the cylindrical structures of cilia and flagella. These microtubule structures are called axonemes. The typical 9+2 arrangement, as shown in Figure 1, is a cylinder comprised of 9 doublet microtubules surrounding a pair of singlet microtubules. The doublet microtubules anchor one end of dynein, the structural attachment, while the other end of the dynein complex interacts cyclically with adjacent doublet microtubules to produce shear or sliding force. In addition to the dyneins, radial spokes attach to the peripheral doublet microtubules and span the space toward the central pair of microtubules. The radial spokes and central