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# MASS QUANTIZATION OF THE NON-EQUILIBRIUM MEAN FIELD TO SELF-INTERACTING PARTICLES

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**Abstract.** Blowup mechanism of a parabolic-elliptic system of cross-diffusion is studied. This system describes the mean field of self-interacting particles, and the blowup solution develops a finite number of singularities like the delta function. Here, it is shown that each re-scaled collapse has the quantized mass. The possibilities of the existence of the remainder term and the formation of multi-collapses of the pre-scaled collapse are also dicsussed.

**Keywords.** Chemotaxis, Free Energy, Blowup, Elliptic-Parabolic System, Self-interacting Particles.

**AMS (MOS) subject classification:** 35K55, 35K57, 92C15, 92D15.

# 1 Introduction

This paper is concerned with the elliptic-parabolic system of cross-diffusion,

$$\left.\begin{array}{l} u_t = \nabla \cdot (\nabla u - u\nabla v) \\ 0 = \Delta v - av + u \end{array}\right\} \quad \text{in} \quad \Omega \times (0,T)$$
$$\frac{\partial u}{\partial \nu} = \frac{\partial v}{\partial \nu} = 0 \quad \text{on} \quad \partial\Omega \times (0,T)$$
$$u|_{t=0} = u_0(x) \quad \text{in} \quad \Omega, \tag{1}$$

where $\Omega \subset \mathbf{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$, $a > 0$ is a constant, and $\nu$ is the outer unit normal vector on $\partial\Omega$.

In the context of mathematical biology, it is proposed by Nagai [18] as a simplified form of the ones given by Jäger and Luckhaus [15], Nanjundiah [23], Keller and Segel [16], and Patlak [26]. Here, $u = u(x,t)$ and $v = v(x,t)$ stand for the density of cellular slime molds and the concentration of chemical substances secreted by themselves, respectively, at the position $x \in \Omega$ and the time $t > 0$.

In this case, the first equation describes the conservation of mass, where the flux of $u$ is given by $\mathcal{F} = -\nabla u + u\nabla v$, as

$$\frac{d}{dt}\int_\omega u = -\int_{\partial\omega} \mathcal{F}\cdot\nu$$