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OPTIMAL CONTROL OF THE SPATIAL MOTION OF A VISCOELASTIC ROD

This paper is dedicated to John Cannon
on the occasion of his 60th birthday

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Abstract. This paper treats two typical optimal control problems for a nonlinearly viscoelastic rod moving in space. In each problem the rod is required to reach a prescribed target consisting of a certain set of positions and velocities. In one case the time is to be minimized and in the other a quadratic cost functional is to be minimized. The admissible controls lie in a bounded set of forces and couples applied to one end of the rod. The governing equations form a quasilinear parabolic-hyperbolic system. The existence of optimal controls depends on suitable compactness results obtained here.
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1 Introduction

We consider a model [1] of a nonlinearly viscoelastic rod moving in 3-dimensional space, taking into account not only longitudinal and transverse motions, but also shear and torsional motion: One may visualize this, discretized, as a chain of hard vertebrae connected by a viscous springy material. This paper is related to the forthcoming [3] much as [9] was related to [2]. The papers [2] and [9] treat the purely longitudinal motion of a straight rod. As in [9], we show that the attainment of optimality for certain control problems is intimately related to the considerations involved in showing the existence of solutions, in particular, to the requirement that a subsequential limit of solutions to some approximating problems should be solutions of a desired limit problem.

For the model we consider, the geometric state at each point of the reference configuration (which we take to be parametrized by $s \in [0,1]$) consists of the position $\mathbf{r} = \mathbf{r}(t,s)$ in 3-dimensional space and the orientation of the