Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 661-678

DIFFUSION IN DEFORMING POROUS MEDIA

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Abstract. We report on some recent progress in the mathematical theory of nonlinear fluid transport and poro-mechanics, specifically, the design, analysis and application of mathematical models for the flow of fluids driven by the coupled pressure and stress distributions within a deforming heterogeneous porous structure. The goal of this work is to develop a set of mathematical models of coupled flow and deformation processes as a basis for fundamental research on the theoretical and numerical modeling and simulation of flow in deforming heterogeneous porous media.
Keywords. Poroelasticity, poro-plasticity, infiltration, degenerate parabolic systems.
AMS (MOS) subject classification: Primary 74F10, 76S05, 74C05; Secondary 35K90, 35K65, 35F25

1 Introduction

Deformation of a saturated porous solid affects the flow of fluid through it, and the fluid pressure contributes to the mechanical behaviour of that structure. Since the load on such a porous structure is supported by both the solid matrix and the fluid in the pores, the coupling of the stress in the solid to the pressure of the fluid plays an essential role. It involves the *dilation* or *contraction* of the deforming matrix and the *pressure gradient* of the diffusing pore fluid. In the classical *consolidation* process, a load is initially shared with the pore fluid, and with time the pore fluid pressure dissipates and the load is increasingly born by the porous solid matrix. The diffusing pore fluid thereby has an important effect on the mechanical response of the matrix. That is, for more slowly applied loads the material response appears less stiff, since the fluid has relatively more time to diffuse away. Conversely, the dilations of the matrix modify the porosity and thereby enhance the fluid flow. The classical Biot model of this process was developed in *soil science*, and it has been refined considerably for the increasingly more demanding needs in engineering and geophysics [17], [44], [63]. The simplest model describes the evolution of the scalar field of *fluid pressure* $p(x,t)$ and the vector field of *solid displacement* $\mathbf{u}(x,t)$ from the position $x \in \Omega$ at time $t > 0$. For a homogeneous and isotropic medium the classical linear *poroelasticity system*