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ANALYTICAL AND NUMERICAL APPROXIMATIONS FOR THE EARLY EXERCISE BOUNDARY FOR AMERICAN PUT OPTIONS

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Abstract. Several new analytical and numerical approximations are provided for the location of the early exercise boundary for the American put option. The most complete approximation is in the form of an integro-differential equation for which an iterative scheme can be proven to converge to the unique solution. The current methods are compared to those recently proposed by other authors. This paper summarizes various parts of our joint work [8-10] with L. Jiang (Shanghai), R. Stamicar (Toronto) and W. Zheng (Irvine).

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1 Statement of the Problem and Previous Results

A European put (call) option gives the purchaser the right, but not the obligation, to sell (buy) a share of an underlying asset at a given price K, the strike, at a given time in the future, T. The payoff at expiry T for a put option can be determined as follows. If the stock price at expiry, $S(T)$, is less than K then the holder can purchase the stock at $S(T)$ and has the right to sell it at K for a payoff of $K - S(T)$. If, on the other hand, $S(T)$ is greater than K the above strategy does not work and if she is holding stock it is preferable to sell it at $S(T)$ in the open market. Thus there is no incentive to exercise the put option and it is worthless in this case. Summarizing, the value of the put option at expiry is

$$p_E(S,T) = \max\ (K - S, 0) \tag{1}$$

The American version of these options allows the holder to exercise them at any time up to expiry, T. Clearly, because the holder has more optionality, the American version is more expensive than the simpler European version. More interestingly from a mathematical viewpoint is the question of determining the early exercise boundary - the asset price at time $t < T, S_f(t)$, below which it is advisable to exercise early.

The Nobel Prize work of Black, Scholes and Merton (see for example [21, Chap. 3]) provides a risk neutral method of pricing these as well as other