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CRITICAL POINT THEORY APPLIED TO A NONLINEAR BIHARMONIC EQUATION

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ABSTRACT: We are concerned with the multiplicity of solutions of the nonlinear biharmonic equation with Dirichlet boundary condition, $\Delta^2 u + c\Delta u = g(u)$, in Ω, where $c \in R$ and Δ^2 denotes the biharmonic operator. We reveal the multiplicity of solutions of the nonlinear biharmonic equation by critical point theory.

Keywords. Dirichlet boundary condition, multiplicity of solutions, critical level, linking theorem, eigenvalue

AMS subject classification: 35J35, 35J40

1 INTRODUCTION

Let Ω be a smooth bounded region in R^n with smooth boundary $\partial\Omega$. Let $\lambda_1 < \lambda_2 \leq \ldots \leq \lambda_k \leq \ldots$ be the eigenvalues of $-\Delta$ with Dirichlet boundary condition in Ω. Let $g : R \to R$ be a differentiable function such that $g(0) = 0$, and

$$g'(\infty) = \lim_{|u|\to\infty} \frac{g(u)}{u} \in R.$$

In this paper we are concerned with the multiple solutions of the nonlinear biharmonic equation with Dirichlet boundary condition

$$\Delta^2 u + c\Delta u = g(u) \qquad \text{in } \Omega, \tag{1.1}$$
$$u = 0, \qquad \Delta u = 0 \qquad \text{on } \partial\Omega,$$

where $c \in R$ and Δ^2 denotes the biharmonic operator. This type problem was studied by Choi and Jung in [6]: The authors proved that problem (1.1) has at least two solutions by the Variation of Linking Theorem under the condition that g is a differentiable function with $g(0) = 0$, $\lambda_i < c < \lambda_{i+1}$, $\lambda_{i+1}(\lambda_{i+1} - c) < \lambda_k(\lambda_k - c) < g'(\infty) < \lambda_{k+1}(\lambda_{k+1} - c), \lambda_{k+m}(\lambda_{k+m} - c) < g'(0) < \lambda_{k+m+1}(\lambda_{k+m+1} - c)$ for $m \geq 1$, and $g'(t) \leq \gamma < \lambda_{k+m+1}(\lambda_{k+m+1} - c)$, $k > i+1$. The nonlinear biharmonic equation with jumping nonlinearity was extensively studied by some authors [7,9,14]. Choi and Jung studied the following problem in [7]

$$\Delta^2 u + c\Delta u = bu^+ + f \qquad \text{in } \Omega, \tag{1.2}$$