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Stability Criteria for Certain Second Order Delay Differential Equations

Baruch Cahlon and Darrell Schmidt

Department of Mathematics and Statistics
Oakland University
Rochester, MI 48309–4485

Abstract. In this paper we study the asymptotic stability of the zero solution of second order linear delay differential equations of the form

$$y''(t) = p_1 y\prime(t) + p_2 y\prime(t-\tau) + q_1 y(t) + q_2 y(t-\tau)$$

where p_1, p_2, q_1, and q_2 are certain constants. Here $\tau > 0$ is a constant. In proving our results we make use of Pontryagin's theory for quasi-polynomials.

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1 Introduction

The aim of this paper is to study the asymptotic stability of the zero solution of the delay differential equation

$$y''(t) = p_1 y\prime(t) + p_2 y\prime(t-\tau) + q_1 y(t) + q_2 y(t-\tau) \tag{1.1}$$

where $\tau > 0$ is a constant and p_1, p_2, q_1, and q_2 are constants. Equations of this type appear in many applications. Delays in human reflexes lead to mathematical models of biological processes whose linearization are of this form [22,18]. In machine tool analysis an important source of instability in the cutting process is the so-called regenerative effect [24]. It is a "past-effect" which has a clear physical explanation. The cutting force depends on the actual and delayed values of the relative displacement of the tool and the workpiece. A simple linearized mathematical model of this phenomenon leads to an equation of the form (1.1). Also stability of chemical reactions lead to such equations (See [23], Chapter 4). Equations of the form of (1.1) can be used as test equations for numerical methods. The authors are not aware of a comprehensive study of this important equation. Earlier work was done by Bhatt and Hsu [3] for special cases of equation (1.1) ($p_2 = 0$ or $q_2 = 0$) using Pontryagin's Theorems. Theorems 3.2 and 3.4 in this paper are rediscoveries of results in [3]. Since our proofs are different and short, we include them for completeness. This equation was studied by Cooke and Grossman [8]