Solutions of the Second Order Nonlinear Impulsive Integro–differential Equations of Mixed Type in Banach Spaces

Li Shan Liu¹, Xiao Yan Zhang¹ and Yeol Je Cho²

¹ Department of Mathematics
Qufu Normal University, Qufu, Shandong 273165, People’s Republic of China
² Department of Mathematics Education
Gyeongsang National University, Chinju 660-701, Korea

Abstract. In this paper, by use of a new comparison result and Mönch’s fixed point theorem, the existence of solutions of initial value problems for nonlinear second order impulsive integro-differential equations of mixed type in Banach spaces is investigated under the weaker conditions.

Keywords. Banach space, measure of noncompactness, initial value problem, impulsive integro-differential equation of mixed type.

AMS (MOS) subject classification: 45J05, 34G20.

1 Introduction

Let \((E, \| \cdot \|)\) be a real Banach space, \(J = [0, a] (a > 0)\) and \(0 < t_1 < t_2 < \cdots < t_m < a\). We consider the following initial value problem (IVP) for nonlinear second order impulsive integro-differential equation of mixed type in \(E\):

\[
\begin{align*}
    u'' &= f(t, u, u', Tu, Su), \quad t \in J, \ t \neq t_k, \\
    \Delta u|_{t=t_k} &= I_k(u(t_k), u'(t_k)), \\
    \Delta u'|_{t=t_k} &= T_k(u(t_k), u'(t_k)), \quad k = 1, 2, \cdots, m, \\
    u(0) &= u_0, \ u'(0) = u_1,
\end{align*}
\]  

where \(f \in C[J \times E \times E \times E \times E, E]\), \(I_k, T_k \in C[E \times E, E] (k = 1, 2, \cdots, m)\), \(u_0, u_1 \in E\),

\[
(Tu)(t) = \int_0^t k(t, s)u(s)ds, \quad (Su)(t) = \int_0^t h(t, s)u(s)ds,
\]

2 The corresponding author: yjchonongae.gsnu.ac.kr (Yeol Je Cho).
1 The first and second authors were supported financially by the National Nature Science Foundation of China (19871048) and the Nature Science Foundation of Shandong Province of China (Z2000A02), and the second author was supported by the Korea Research Foundation Grant (KRF-2001-005-D00002).