

Solutions of the Second Order Nonlinear Impulsive Integro-differential Equations of Mixed Type in Banach Spaces

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Abstract. In this paper, by use of a new comparison result and Mönch's fixed point theorem, the existence of solutions of initial value problems for nonlinear second order impulsive integro-differential equations of mixed type in Banach spaces is investigated under the weaker conditions.

Keywords. Banach space, measure of noncompactness, initial value problem, impulsive integro-differential equation of mixed type.

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1 Introduction

Let $(E, \|\cdot\|)$ be a real Banach space, $J = [0, a]$ ($a > 0$) and $0 < t_1 < t_2 < \dots < t_m < a$. We consider the following initial value problem (IVP) for nonlinear second order impulsive integro-differential equation of mixed type in E :

$$\begin{cases} u'' = f(t, u, u', Tu, Su), & t \in J, t \neq t_k, \\ \Delta u|_{t=t_k} = I_k(u(t_k), u'(t_k)), \\ \Delta u'|_{t=t_k} = \bar{I}_k(u(t_k), u'(t_k)), & k = 1, 2, \dots, m, \\ u(0) = u_0, u'(0) = u_1, \end{cases} \quad (1.1)$$

where $f \in C[J \times E \times E \times E \times E, E]$, $I_k, \bar{I}_k \in C[E \times E, E]$ ($k = 1, 2, \dots, m$), $u_0, u_1 \in E$,

$$(Tu)(t) = \int_0^t k(t, s)u(s)ds, \quad (Su)(t) = \int_0^a h(t, s)u(s)ds, \quad (1.2)$$

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