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UNIFIED APPROACH TO MONOTONE ITERATIVE TECHNIQUE FOR SEMILINEAR PARABOLIC PROBLEMS

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Abstract. A unified setting for the monotone iterative technique is considered relative to semilinear parabolic problems when the nonlinear term involved admits a splitting of the difference of two monotone functions. This approach includes several interesting results in one framework.
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1 Introduction

Recently [3], [4], a unified approach to the monotone iterative technique [1], [2], [5] is developed to semilinear and quasilinear elliptic problems. The main idea is to consider the situation when the nonlinear term involved admits a splitting of a difference of two monotone functions or equivalently, of a sum of two functions, one is increasing and the other is decreasing. This setting includes several known results as well as provides some interesting new ones. In this paper, we extend this approach to semilinear parabolic problems.

2 Comparison Result

Let $\Omega \subset R^N$ be a bounded domain with Lipschitz boundary $\partial\Omega$, $Q = \Omega \times (0,T)$ and $\Gamma = \partial\Omega \times (0,T)$, $T > 0$. Consider the following initial boundary value problem (IBVP)

$$\left[\begin{array}{ll} u_t + Lu = f(x,t,u) + g(x,t,u) & \text{in } Q, \\ u = 0 \ \text{ on } \ \Gamma \text{ and} & u(x,0) = 0 \text{ in } \Omega, \end{array} \right. \tag{1}$$

where L denotes the second order partial differential operator in the divergence form

$$Lu = -\sum_{i,j=1}^{N} \frac{\partial}{\partial x_i}(a_{i,j}(x,t)\frac{\partial u}{\partial x_j}) + c(x,t)u,$$