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BLOW-UP OF POSITIVE SOLUTIONS OF A SEMILINEAR PARABOLIC EQUATION WITH A GRADIENT TERM

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Abstract. We study the blow-up behavior (in time and space) of positive solutions of a semilinear parabolic equation with a gradient term. Our main result is a sharp estimate for the spatial blow-up profile of radially decreasing solutions on a ball. This result illustrates the influence of the gradient term on the profile.
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1 Introduction

In this paper we will study the initial-boundary value problem

$$u_t = \Delta u - |\nabla u|^q + u^p \qquad \text{in } \Omega \times (0,T), \tag{1.1}$$
$$u(x,t) = 0 \qquad \text{if } (x,t) \in \partial\Omega \times (0,T), \tag{1.2}$$
$$u(x,0) = u_0(x) \qquad \text{if } x \in \Omega, \tag{1.3}$$

where $p, q > 1$, Ω is a bounded domain in $\mathbb{R}^n$, with C^2 boundary, and $u_0 \geq 0$.

This problem was introduced in [3] and it was studied later in [5, 7–11, 14], for instance. The main issue in those works was to determine for which p and q blow-up in finite time (in the L^∞-norm) may occur. It turns out (see [14]) that it occurs if and only if $p > q$. Equation (1.1) in $\mathbb{R}^n$ was considered in [1, 14, 15] from a similar point of view. In this case, blow-up in finite time is also known to occur when $p > q$ (see [14]) but unbounded solutions always exist (see [15]).

The main aim of this paper is to show that if $\Omega = B_R = \{x \in \mathbb{R}^n : |x| < R\}$ and $u_0 = u_0(r)$, $u_0'(r) \leq 0$, then the estimate

$$u(r,t) \leq Cr^{-\alpha}, \quad (r,t) \in (0,R] \times [0,T) \tag{1.4}$$

holds for any $\alpha > 2/(p-1)$ if $q \in (1, 2p/(p+1))$ and for any $\alpha > q/(p-q)$ if $q \in [2p/(p+1), p)$. Let

$$\alpha_0 := \inf\{\alpha > 0 : (1.4) \text{ holds for some } C = C(\alpha) > 0\}. \tag{1.5}$$