Dynamics of Continuous, Discrete and Impulsive Systems
Series **A:** *Mathematical Analysis* **10 (2003)** 491-513

ON THE UPPER SEMICONTINUITY OF COCYCLE ATTRACTORS FOR NON-AUTONOMOUS AND RANDOM DYNAMICAL SYSTEMS

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Abstract. In this paper we prove an upper semicontinuity result for perturbations of cocycle attractors. In particular, we study relationship between non-autonomous and global attractors. In this sense, we show that the concept of a cocycle attractor is a sensible generalization to non-autonomous and random dynamical systems of that of the global attractor.

1 Introduction

Some qualitative properties of the asymptotic behaviour of nonlinear autonomous differential equations can be obtained from the study of their global attractors: these are compact attracting sets that are invariant for the semigroup associated to the equation (Hale [15], Temam [26]). The theory of such global attractors is now well developed, both in the finite and infinite-dimensional cases.

Important new difficulties appear when the differential equation is non-autonomous. In this case, the semigroup becomes a process, that is, an operator which depends on two parameters, since the dependence on the initial time is as important as that on the final time (Sell [25]). When the non-autonomous terms are periodic or quasi-periodic, the standard concept of a global attractor can be used for these situations (Sell [25], Chepyzhov and Vishik [6]). However, important changes must be introduced when we deal with general non-autonomous terms. Chepyzhov and Vishik [6] define the concepts of *the kernel* and *the kernel sections,* the latter similar to the notion of a cocycle or pullback attractor defined in Cheban et al. [5]. In our opinion, this is the right way to define "an attractor" for a general non-autonomous differential equation. The attractor in this situation is a time-dependent family of compact sets that is invariant with respect to the cocycle and attracting 'from $-\infty$' (see Definition 4).

The main result in this paper (a generalisation of that in Caraballo et al. [4] which treats only cocycle perturbations of semigroups) guarantees the upper semicontinuity of a cocycle attractor with respect to small perturbations