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ATTRACTIVE PERIODIC ORBITS FOR DISCRETE MONOTONE DYNAMICAL SYSTEMS ARISING FROM DELAYED NEURAL NETWORKS

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Abstract. We consider the discrete-time system

$$\begin{cases} x(n) &= \beta x(n-1) + f(y(n-k)), \\ y(n) &= \beta y(n-1) + f(x(n-k)), \end{cases} \quad n \in N$$

describing the dynamic interaction of two identical neurons, where $\beta \in (0,1)$ is the internal decay rate, f is the signal transmission function and k is the signal transmission delay. We obtain the existence and attractivity of a k-periodic orbit by using the contractive map principle and a singular perturbation method recently developed by H. O. Walther. This is in contrast to the continuous case (a delay differential system) where no attractive periodic orbit can occur due to the monotonicity of the associated semiflow.

Keywords. Attracitivity, Periodic orbit, Discrete monotone dynamical systems, Neural networks, Delay.
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1 Introduction

We consider the following nonlinear discrete-time system

$$\begin{cases} x(n) &= \beta x(n-1) + f(y(n-k)), \\ y(n) &= \beta y(n-1) + f(x(n-k)), \end{cases} \tag{1.1}$$

where $n \in N$ (the set of all positive integers), $\beta \in (0,1)$, $k \geq 1$ is a fixed integer, f is a nonlinear function. System (1.1) describes the evolution of a discrete-time network of two identical neurons with excitatory interactions, where β is the internal decay rate, f is the signal transmission function, and k is the signal transmission delay. See [11] for general backgrounds on delayed neural networks, and [12, 13, 14] for related discrete-time networks.

A basic and important signal transmission function is the McCulloch-Pitts nonlinearity given by

$$f(x) = \begin{cases} 1 & x \geq 0, \\ -1 & x < 0. \end{cases} \tag{1.2}$$