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MICROBIAL COMPETITION FOR NUTRIENT IN A $3D$ FLOW REACTOR

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Abstract. We study the dynamics of a mathematical model of one or more bacterial strains competing for a growth limiting substrate in a cylindrical flow reactor. In previous work, the authors have investigated the case where bacteria can also colonize the lateral boundary surface of the cylinder forming a biofilm. In order to better understand the effects of wall colonization, it is important to have a reference model with which one can compare the dynamics. Our purpose here is to provide such a reference model where wall growth is not allowed and to study its dynamics.

Keywords. competition, flow reactor, coexistence, steady state, parabolic system.
AMS (MOS) subject classification: 92A15, 35K55, 34B25.

1 Introduction

In recent work [2, 3, 4, 8, 7], a mathematical model of microbial growth and competition in a flow reactor in which bacteria are assumed to colonize the wall of the reactor has been studied. The model assumes that bacteria compete for nutrient as well as for a limited number of wall-attachment sites. In order to understand the effects of competition for wall-attachment sites on an organisms ability to colonize the flow reactor and to be a successful competitor in mixed culture, it is important to first understand the model dynamics in the absence of wall growth. It is the purpose of this paper to provide this baseline comparison for the three-dimensional cylindrical flow reactor. The one-dimensional case was treated in [1, 12, 9].

We provide sharp conditions for survival of a single strain in the bioreactor. Its growth rate must exceed loss due to cell death and dilution when substrate levels match those in the feed. When this condition holds, we establish the existence of a steady state solution in which that strain alone