Dynamics of Continuous, Discrete and Impulsive Systems
Series **B:** *Applications & Algorithms* **10 (2003)** 469-480

WAVE-PROPAGATION DYNAMICS IN AN ANISOTROPIC EXCITABLE MEDIUM

John C. Clements[1], Clyde J. Clements[2] and B. Milan Horáček[2]

[1] Department of Mathematics & Statistics
[2] Department of Physiology & Biophysics
Dalhousie University, Halifax, Nova Scotia B3H 3J5, Canada

Abstract. Bidomain models for simulating the propagation of electrical activation in the heart treat the myocardium as an anisotropic excitable medium with conductivity coefficients $\sigma_{\ell,t}^{i,e}$, where i and e refer to intracellular and extracellular space, and ℓ and t indicate whether conductivity is along (ℓ) or transverse (t) to the local fibre direction. These models are made computationally tractable by the *equal anisotropy ratio assumption*, which states that $\sigma_\ell^e/\sigma_\ell^i = \sigma_t^e/\sigma_t^i = k$, for some scalar constant k. Although it is doubtful that this assumption is valid, it has been the only means of reducing the complex coupled systems of nonlinear partial differential equations to a single reaction-diffusion problem. By introducing a simple perturbation argument, we achieved an equivalent reduction—with a de-coupling tensor expressed in terms of the harmonic means $\sigma_{\ell,t}^{i,e}\sigma_{\ell,t}^{i,e}/(\sigma_{\ell,t}^{i,e} + \sigma_{\ell,t}^{i,e})$ of the conductivity parameters—thus preserving the critical information conveyed by conductivity parameters without resorting to the assumption regarding their ratios. Numerical simulations in a realistic tissue volume were performed to assess the consequences of this alternate formulation.
Keywords. anisotropic wave propagation, excitable media, cardiac tissue
AMS (MOS) subject classification: Primary 35K60, 92C05; Secondary 35Q80, 92C30

1 Introduction

Bidomain models of the propagation of electrical activation in the anisotropic cardiac tissue have been developed in the form of coupled systems of ordinary and partial differential equations [2, 4, 5, 6, 8, 10]. In these models, the elliptic partial differential equations arise from the laws of conservation of current, a nonlinear parabolic equation describes propagation, and ordinary differential equations describe the transmembrane ionic currents of cardiac cells. The anisotropy in the ventricular wall is usually defined by the transmural rotation of the myocardial fibres.

In section 2, the derivation of the bidomain model is given. Bidomain models treat the myocardium as an anisotropic region, H, consisting of two interpenetrating media (intracellular and extracellular) which are everywhere connected via a continuous cardiac membrane. The electric potentials within