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TRAVELLING WAVES IN A SIMPLE CHEMICAL SYSTEM WITH FRACTIONAL ORDER AUTOCATALYSIS AND DECAY: COMPARISON WITH ORDERS ABOVE UNITY

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Abstract. We consider a coupled system of reaction-diffusion equations arising as a model of chemical autocatalysis with decay, where the orders of autocatalysis and decay, m and n respectively, are fractional ($0 < m, n < 1$). This case is quite different to when $m, n \geqslant 1$ [10], and we make a comparison of travelling waves in the two cases. We show that, in the case $0 < m, n < 1$, travelling waves can only exist if $n < m$, and that the wave of autocatalyst concentration must have finite support. The existence of travelling waves also requires that a parameter k (measuring the relative strength of decay to autocatalysis) be sufficiently small. We can derive an upper bound for k, but numerical results demonstrate that the actual critical value of k is significantly lower. We also present results illustrating how the wave speed c, the size of the support of the autocatalyst wave, and the constant concentration behind the reactant wave all depend upon k, both using numerics, and the method of matched asymptotics as $k \to 0^+$.

Keywords. Reaction-diffusion, travelling waves, autocatalysis, decay, fractional order.

1 Introduction

This paper is concerned with the analysis of a model, isothermal, autocatalytic, chemical reaction scheme with decay, where the orders of both the autocatalytic step and the decay step are less than unity. Such a scheme can be represented by the chemical reaction equations

$$\begin{array}{lllll} A & \to & B & \text{rate } k_1 a b^m, & \text{(autocatalytic step)} \\ B & \to & C & \text{rate } k_2 b^n, & \text{(decay step)} \end{array}$$

where A is the reactant, B is the autocatalyst and C is an inert product, with a, b, c the respective concentrations, k_1, k_2 are rate constants, and $0 < m, n < 1$ are the orders of the autocatalytic and decay steps respectively.

The nondimensionalised coupled reaction-diffusion system representing the scheme is

$$\frac{\partial \alpha}{\partial t} = \frac{\partial^2 \alpha}{\partial x^2} - [\alpha\beta^m]^+, \tag{1a}$$

$$\frac{\partial \beta}{\partial t} = \frac{\partial^2 \beta}{\partial x^2} + [\alpha\beta^m]^+ - k[\beta^n]^+, \tag{1b}$$