

TRAVELLING WAVES IN A SIMPLE CHEMICAL SYSTEM WITH FRACTIONAL ORDER AUTOCATALYSIS AND DECAY: COMPARISON WITH ORDERS ABOVE UNITY

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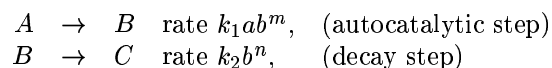
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Abstract. We consider a coupled system of reaction-diffusion equations arising as a model of chemical autocatalysis with decay, where the orders of autocatalysis and decay, m and n respectively, are fractional ($0 < m, n < 1$). This case is quite different to when $m, n \geq 1$ [10], and we make a comparison of travelling waves in the two cases. We show that, in the case $0 < m, n < 1$, travelling waves can only exist if $n < m$, and that the wave of autocatalyst concentration must have finite support. The existence of travelling waves also requires that a parameter k (measuring the relative strength of decay to autocatalysis) be sufficiently small. We can derive an upper bound for k , but numerical results demonstrate that the actual critical value of k is significantly lower. We also present results illustrating how the wave speed c , the size of the support of the autocatalyst wave, and the constant concentration behind the reactant wave all depend upon k , both using numerics, and the method of matched asymptotics as $k \rightarrow 0^+$.

Keywords. Reaction-diffusion, travelling waves, autocatalysis, decay, fractional order.

1 Introduction

This paper is concerned with the analysis of a model, isothermal, autocatalytic, chemical reaction scheme with decay, where the orders of both the autocatalytic step and the decay step are less than unity. Such a scheme can be represented by the chemical reaction equations



where A is the reactant, B is the autocatalyst and C is an inert product, with a, b, c the respective concentrations, k_1, k_2 are rate constants, and $0 < m, n < 1$ are the orders of the autocatalytic and decay steps respectively.

The nondimensionalised coupled reaction-diffusion system representing the scheme is

$$\frac{\partial \alpha}{\partial t} = \frac{\partial^2 \alpha}{\partial x^2} - [\alpha \beta^m]^+, \quad (1a)$$

$$\frac{\partial \beta}{\partial t} = \frac{\partial^2 \beta}{\partial x^2} + [\alpha \beta^m]^+ - k[\beta^n]^+, \quad (1b)$$